TIME VALUE OF MONEY

★ Interest

- Rent paid for using money
- From lender → gain / profit
  From borrower → cost

- Rate at which interest is paid / earned is interest rate, usually expressed per year (% per annum)

- Lender's Viewpoint

- Either spend money now (consume goods) or hold money for future use or lend money (for interest)

- Factors affecting interest rate include risk of borrower not repaying, expected ROI and inflation effects.

- Borrower's Viewpoint

- Must accept terms set by lender, else bear consequences. They borrow to satisfy present needs.

- Factors affecting interest rate include ensuring that for personal use, interest < immediate satisfaction for business use, interest < expected profit.

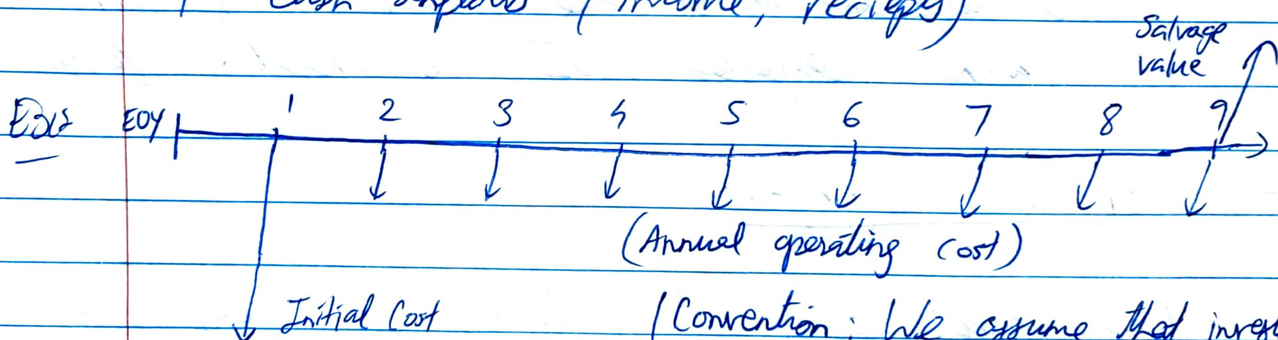
☆ Time Value of Money

- Money today ≠ Same amount in future
- Money has time value, it can earn interest, inflation can reduce value too.
- Measured using interest rate.
- Inflation helps borrowers, hurts lenders
  Deflation helps lenders, hurts borrowers (money becomes more valuable)

☆ Cash Flow Diagram

- Visual model to represent financial problems, showing amount and timing of cash flows.

- Time shown on horizontal line.
  ↓ cash ~~of~~ outflows (cost, payments)
  ↑ cash inflows (income, reciepts)



(Convention: We assume that investments are made at the end of each period in CFD)

☆ Simple Interest

- Interest ~~pa~~ calculated only on principal (P)
- Paid once at end of loan period.

$$SI = P \cdot i \cdot n$$

$$\left[F = P + (P \cdot i \cdot n) \right]$$

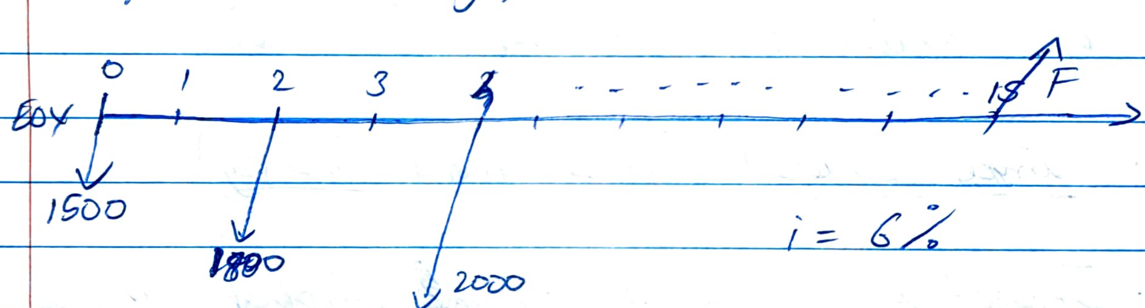
- Interest earned every year is constant (F = final amount)

☆ Compound Interest

- Interest is added to principal, earning interest on interest.
- Interest increases every year. (Exponential)

$$\boxed{F = P(1+i)^n}$$

Ex: If \$1500 is invested now, \$1800 two years from now, and \$2000 four years from now at an interest rate of 6% compounded annually, what will be total amount in 15 years?



$$F = 1500(1+i)^{15} + 1800(1+i)^{13} + 2000(1+i)^{11}$$
$$= 11228.5$$

TIME VALUE OF MONEY

- Money has a time value because it can earn more money over time (earning power), also purchasing power changes over time (inflation)

- Measured in terms of INTEREST RATE which is the cost of money — Cost to Borrower and Earning to the Lender.

- Interest is the rental value of money → growth of capital per unit time

☆ • Simple Interest (interest grows linearly)

- Interest is calculated only on original principle.

$$SI = P \times i \times n$$

$$F = P + I = P(1 + in)$$ (Final amount)

where
P → principal
i → interest rate per period
N → no. of periods

☆ Compound Interest (interest grows exponentially)

- Interest is calculated on principal and previously-earned interest

$$CI = P(1+i)^n$$

→ F (future value)

- Discounting (Present Worth) : $\boxed{P = \frac{F}{(1+i)^n}}$

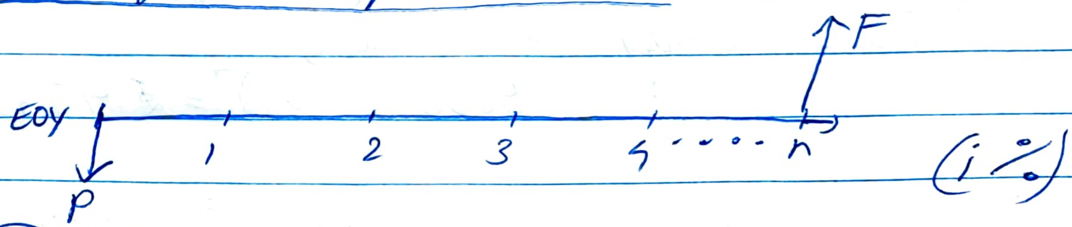
(If you want future amount F, how much should you deposit today?)

Note: Economic equivalence means:
Money at different times can be compared using interest rate

Eg: ₹1000 kept in locker → remains ₹1000
₹1000 in bank @10% → grows over time.
That earning power can be used for equal annual withdrawals (like ₹500/year) or smaller initial deposit.

Note: P → principal amount
n → no. of interest periods
i → interest rate
F → future amount at end of year n
A → equal amount deposited at end of each interest period
G → uniform amount to be added/subtracted period after period to/from amount of deposits A1 at end of that period.

— Single Payment: Compound Amount (Find F, given P)



$\boxed{F = P(1+i)^n} = P\left(\frac{F}{P}, i, n\right)$

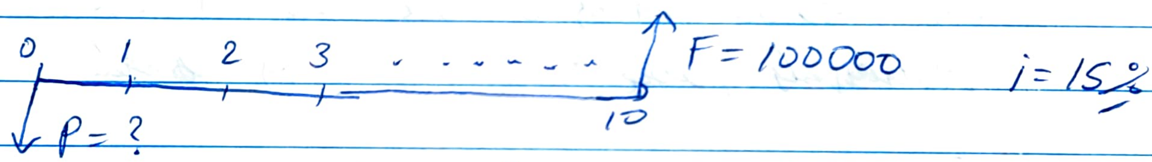
Ex:- Person deposits amount of ₹20000 (P) into an account bearing (i) of 18% compounded annually for 10 years (n). Find maturity value after 10 years (F)

$$F = P(1+i)^n = 20000(1+0.18)^{10} = ₹1,04,680$$

— <u>Single Payment: Present Worth Amount</u> (Find P, given F)

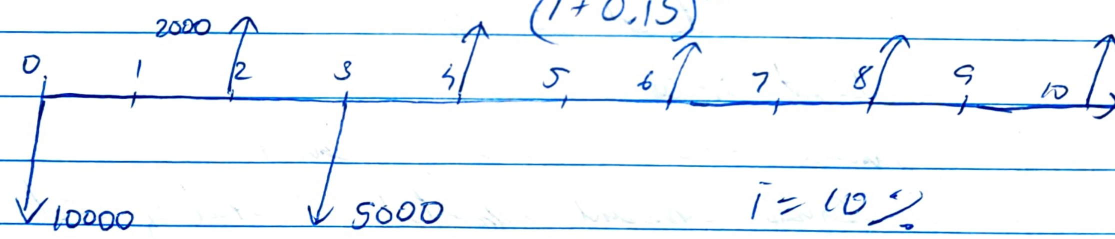
$$P = \frac{F}{(1+i)^n} = F\left(\frac{P}{F}, i, n\right)$$

Ex:-



$$P = \frac{100000}{(1+0.15)^{10}} = ₹24,718$$

Ex:-



To find future value of deposits at year 10.

(i) ₹10000 deposited at year 0 for 10 years,

$$F = 10000(1+0.1)^{10} = ₹25937$$

(ii) ₹5000 deposited for 7 years,

$$F = 5000(1+0.1)^{7} = ₹9743.5$$

∴ Total future value of deposits = ₹35680.5

To find future value of withdrawals, (compounded forward)

Since each withdrawal amount leaves the account and stops earning interest, we are trying to determine the lost growth — how much that withdrawn amount would have become by year 10.

years to grow

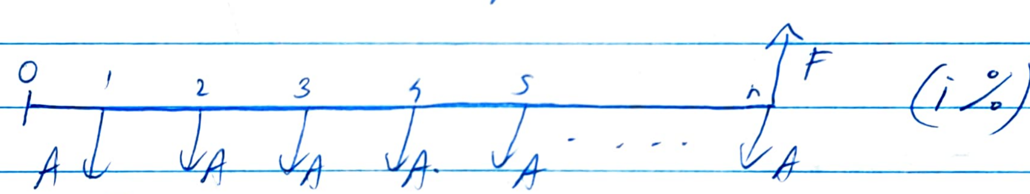
$\therefore$ Total future values of withdrawals $= 2000(1+i)^8 + 2000(1+i)^6 + 2000(1+i)^4 + 2000(1+i)^2 + 2000(1+i)^0$

$= ₹15,178.6$

$\therefore$ Balance left in account = Deposit − Withdrawal

$= ₹20502.5$

– Equal Payment Series: Compound Amount (Find F, given A)

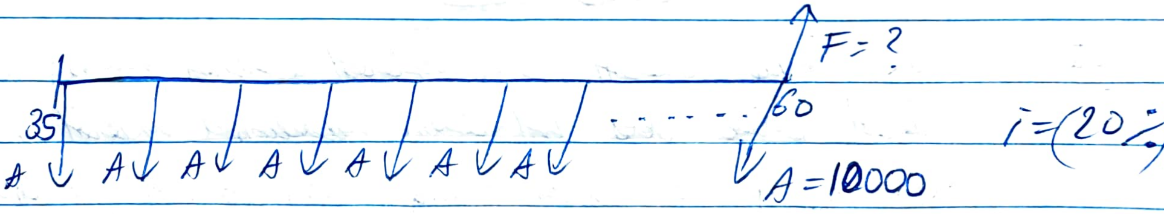


$$\boxed{F = A\left[\frac{(1+i)^n - 1}{i}\right]} = A\left(\frac{F}{A}, i, n\right)$$

– Equal Payment Series: Sinking Fund (Find A, given F)

$$\boxed{A = F\left[\frac{i}{(1+i)^n - 1}\right]} = F\left(\frac{A}{F}, i, n\right)$$

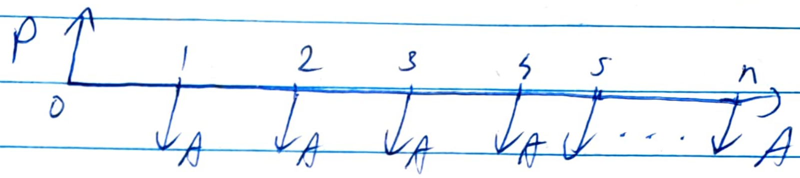
Ex:



$n = 25$

$$F = 10000\left[\frac{(1+0.2)^{25} - 1}{0.2}\right] = ₹4719810$$

- **Equal Payment Series: Present Worth (Find P, given A)**



$$\boxed{P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]} = A\left(\frac{P}{A}, i, n\right)$$

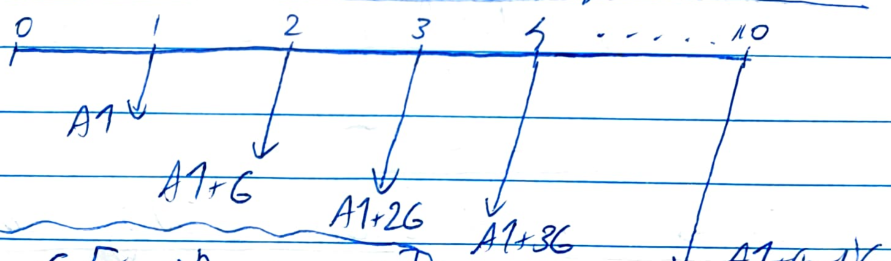
- **Equal Payment Series: Capital Recovery Amount**

(Find A, given P)

$$\boxed{A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]} = P\left(\frac{A}{P}, i, n\right)$$

~~Equal~~

- **Uniform Gradient Series: Annual Equivalent Amount**



$$A = A1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$
$$= A1 + G(A/G, i, n)$$

G → equal increment

Note: With simple interest, the interest earned during each interest period does not earn additional interest in the remaining periods, even though you do not withdraw it.

- Additional interest earned with CI over SI,

$$\Delta I = P\left[(1+i)^n - (1+in)\right]$$

Note For equal payment series, fund invested at the end of x^{th} period will be worth $A(1+i)^{N-x}$ at the end of N periods
So on until the A dollars we invest at the end of N^{th} period will be worth $A(1+i)^{N-N} = A$ dollars

$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \dots + A(1+i) + A$$

$$F = A + A(1+i) + A(1+i)^2 + \dots + A(1+i)^{N-1} \quad -①$$

Multiplying by $(1+i)$,

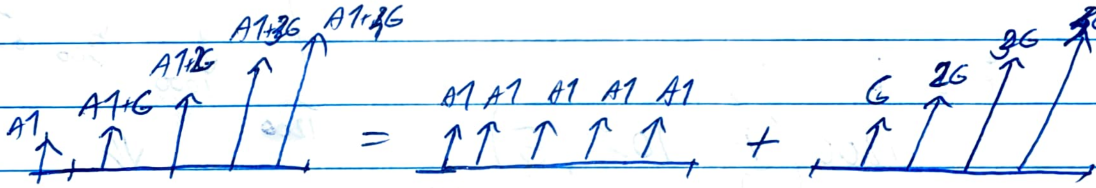
$$F(1+i) = A(1+i) + A(1+i)^2 + \dots + A(1+i)^N \quad -②$$

② − ①,

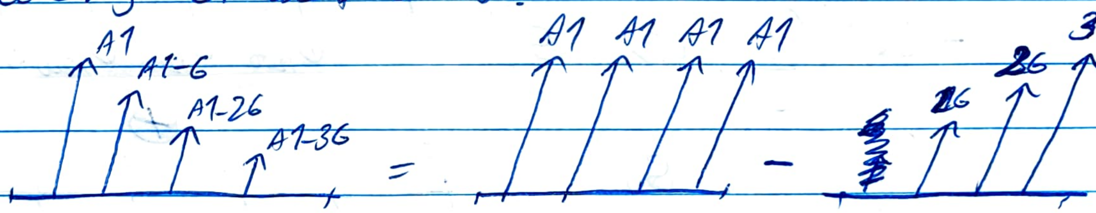
$$F(1+i) - F = -A + A(1+i)^N$$

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A\left(\frac{F}{A}, i, N\right)$$

Note Increasing Gradient Series:



Decreasing Gradient Series:



Note: Gradient Series: Present-Worth Factor

$$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right] = G(P/G, i, N)$$

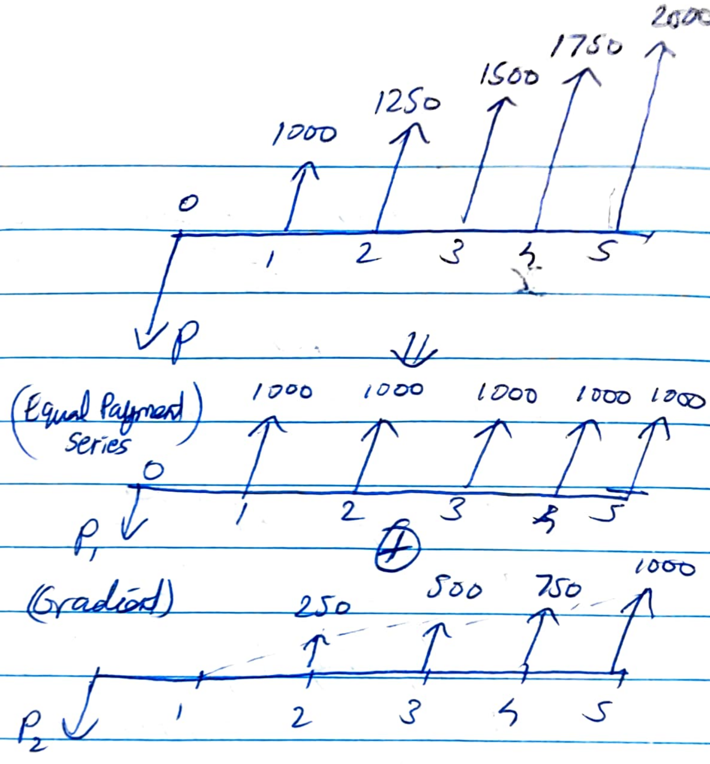
Ex:

$A_1 = 1000$

$G = 250$

$i = 12\%$

$N = 5$



$P = P_1 + P_2$

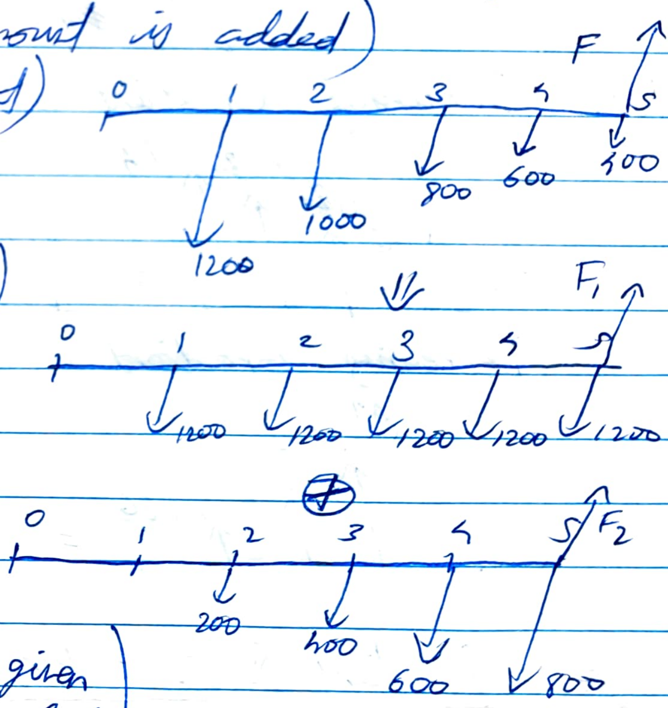
$= A_1 \left(\frac{P}{A}, 12\%, 5\right)$

$+ G \left(\frac{P}{G}, 12\%, 5\right)$

$= 5204$

(Here, for gradient N=5 as gradient series starts ~~when~~ before gradient amount is added)

Ex: (Declining Linear Gradient)



$F = F_1 - F_2$

$= 1200 \left(\frac{F}{A}, 10\%, 5\right)$

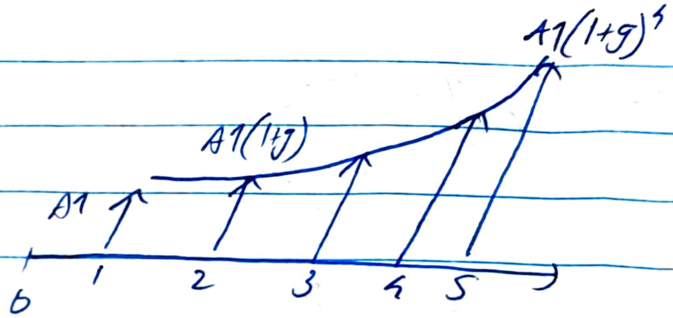
$- 200 \left(\frac{P}{G}, 10\%, 5\right) \left(\frac{F}{P}, 10\%, 5\right)$

$= 5115$

(Find P given G then find F given P)

Note

Geometric Gradient Series



$$\boxed{A_n = A_1(1+g)^{n-1}}$$

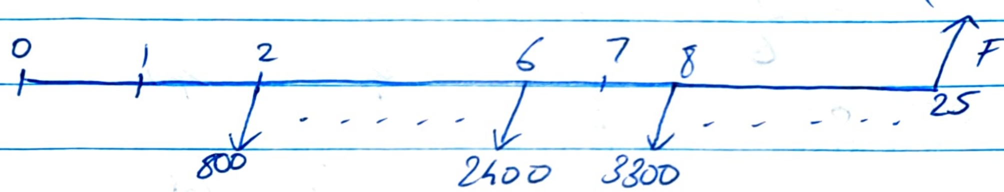
$$\boxed{P_n = A_n(1+i)^{-n}} = A_1(1+g)^{n-1}(1+i)^{-n}$$

$$P = \begin{cases} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] & \text{if } i \neq g \\ \dfrac{NA_1}{1+i} & \text{if } i = g \end{cases}$$

$$P = A_1\left(\frac{P}{A_1}, g, i, N\right)$$

Exer ①

[Refer PPT for question]

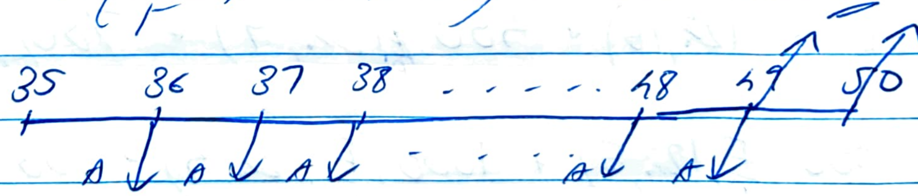


$i = 18\%$, $n = 25$

$$F = 800\left(\frac{F}{P}, 18\%, 23\right) + 2400\left(\frac{F}{P}, 18\%, 19\right) + 3300\left(\frac{F}{P}, 18\%, 17\right) = ₹146737.6$$

$$A = F\left(\frac{A}{F}, 18\%, 25\right) = ₹428.47$$

Exer ②



$A = ₹19760$, $n = 14$, $i = 8\%$

$$F_{49} = A\left(\frac{F}{A}, 8\%, 14\right) = ₹478488.4$$

$$F_{50} = F_{49}(1+i)^1 = ₹516767.47$$

(just interest)

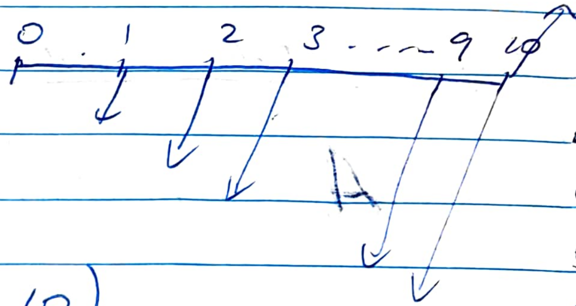
(1) Ex

$A1 = ₹4000$
$G = ₹500$
$n = 10$
$i = 15\%$

$A = A1 + G\left(\frac{A}{G}, 15\%, 10\right)$

$= ₹5691.60$

$F = A\left(\frac{F}{A}, 15\%, 10\right) = ₹1,15,560.216$



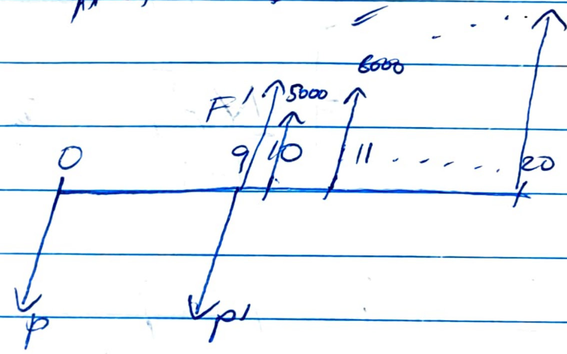
(2) Ex

$P' = 5000\left(\frac{P}{A}, 12\%, 11\right) + 1000\left(\frac{P}{G}, 12\%, 11\right)$

$= ₹52819 = F'$

$P = F'\left(\frac{P}{F}, 12\%, 9\right)$

$= ₹19046.53$

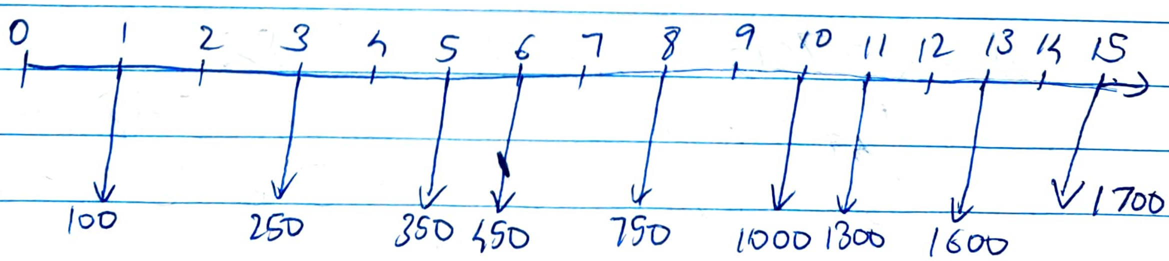


(we start the year before gradient starts)

(5) Ex

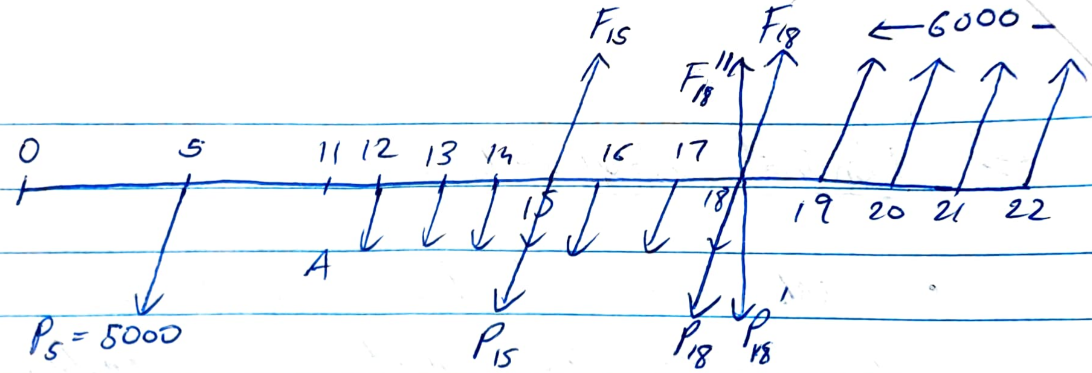
$n = 15, \quad i = 12\%$

$F = 100\left(\frac{F}{P}, 12\%, 14\right) + 250\left(\frac{F}{P}, 12\%, 12\right) + 350\left(\frac{F}{P}, 12\%, 10\right) + 450\left(\frac{F}{P}, 12\%, 9\right) + 750\left(\frac{F}{P}, 12\%, 7\right) + 1000\left(\frac{F}{P}, 12\%, 5\right) + 1300\left(\frac{F}{P}, 12\%, 4\right) + 1600\left(\frac{F}{P}, 12\%, 2\right) + 1700$



③ Ex8

(Boy)



(Grandpa) $P_5 = 5000$

$F_{15} = 5000\left(\frac{F}{P}, 6\%, 10\right) = 8955 = P_{15}$ (matures, reinvested)

$P_{18} = F_{18} = 8955\left(\frac{F}{P}, 6.5\%, 3\right) = 10817.18$

(What we have)

Expected $P_{18}' = 6000\left(\frac{P}{A}, 6.5\%, 4\right) = 20554.8$

(to get 6000 yearly) (What we want)

Remaining = Want − Have = 9737.6

$F_{18}'' = 9737.6$

(Parents) $A = F_{18}''\left(\frac{A}{F}, 6.5\%, 7\right) = ₤1142.5$

(extra to invest, from now)

(Goyim)

Nominal Interest Rate (r)

- "Stated" or "advertised" interest rate for a full year which ignores the effect of compounding.

Effective Interest Rate (i_{eff})

- Actual interest rate you end up paying / earning over a full year, which is compounded (interest on top of interest)

$$\boxed{i_{eff} = \left(1 + \frac{r}{ck}\right)^{c} - 1}$$

$c \rightarrow$ no. of compounding periods per payment period

$k \rightarrow$ no. of payments periods per year.

Ex:
- 2% per month
  $r = 24\%$ /year
- 4% per semi-annual
  $r = 8\%$ /year
- 5% per quarter
  $r = 20\%$ /year

- 6% (r) compounded semi-annually
  ~~=~~ 3% per semi-annual period
- 9% compounded quarterly (r)
  = 2.25% per quarter

Ex:
- 0.5% per month
  $r = 0.5 \times 12 = 6\%$ compounded monthly
  $i_{eff} = \left(1 + \frac{0.06}{12 \times 1}\right)^{12} - 1 = 6.17\%$ /year

- 4.5% per semi-annual
  $r = 9\%$ compounded semi-annually
  $i_{eff} = \left(1 + \frac{0.09}{2 \times 1}\right)^{2} - 1 = 9.2\%$ /year

2% per quarter

$r = 8\%$ compounded quarterly

$$i_{eff} = \left(1 + \frac{0.08}{4 \times 1}\right)^4 - 1 = 8.24\% \text{ /year}$$

Ex:- Find i_{eff} per quarter at $r = 8\%$ compounded:

(quarter/year)

(a) quarterly $\Rightarrow$ $c = 1$, $k = 4$, $i_{eff} = \left(1 + \frac{0.08}{1 \times 4}\right)^1 - 1 =$

(quarter/quarter)

(b) monthly $\Rightarrow$ $c = 3$, $k = 4$, $i_{eff} = \left(1 + \frac{0.08}{3 \times 4}\right)^3 - 1 =$

(months/quarter)

(c) weekly $\Rightarrow$ $c = \frac{52}{4} = 13$, $k = 4$, $i_{eff} = \left(1 + \frac{0.08}{13 \times 4}\right)^{13} - 1 =$

(d) daily $\Rightarrow$ $c = \frac{365}{4} = 91.25$, $k = 4$, $i_{eff} = \left(1 + \frac{0.08}{91.25 \times 4}\right)^{91.25} - 1$

Ex-(7) $r = 12\%$, $n = 5$, $A = 1200$

$$i_{eff} = \left(1 + \frac{0.12}{12 \times 1}\right)^{12} = 12.68\%$$

$$F = A\left(\frac{F}{A}, 12.68\%, 5\right) = 7728$$

Ex-(3) Payment Period, PP = 6 months = twice in a year (semi-annual)

$n = 10$ (semi-annuals in 5 years)

$k = 2$ (semi-annuals (PP) in a year)

(a) $r = 12\%$ compounded semi-annually

$$i_{eff} = \left(1 + \frac{0.12}{1 \times 2}\right)^1 - 1 = 0.06 = 6\%$$

($c = 1$, $\Rightarrow$ semi-annuals per semi-annual)

$$F = 6000\left(\frac{F}{A}, 6\%, 10\right) = ₹79056$$

(b) $c = \frac{1}{2}$ (years per semi-annual)

$r = 12\%$ compounded annually

$$i_{eff} = \left(1 + \frac{0.12}{\frac{1}{2} \times 2}\right)^{\frac{1}{2}} - 1 = 5.8\%$$

$$F = 6000\left(\frac{F}{A}, 5.8\%, 10\right) = ₹78345$$

(c) $c = 2$ (quarters per semi-annual)

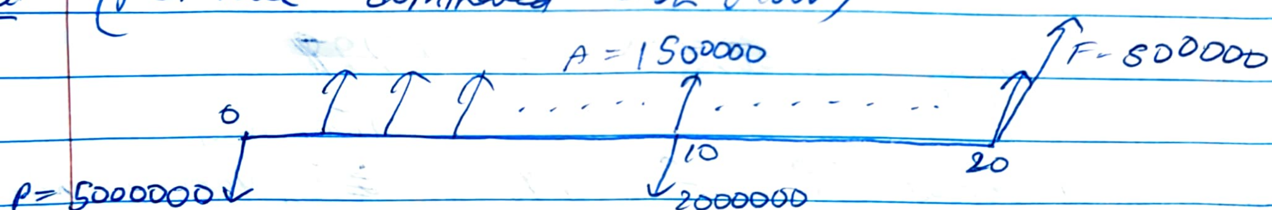
$r = 12\%$ compounded quarter

$$i_{eff} = \left(1 + \frac{0.12}{2 \times 2}\right)^{2} - 1 = 6.09\%$$

$$F = 6000\left(\frac{F}{A}, 6.09\%, 10\right) = ₹79419.83$$

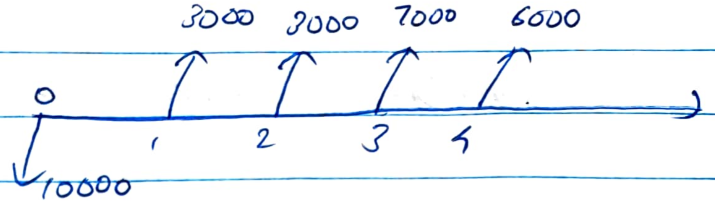
☆ Present Worth

① Ex — (Revenue - dominated cash flow)



$$\text{Net Present Worth} = -5000000 - 2000000\left(\frac{P}{F}, 20\%, 10\right)$$
$$+ 1500000\left(\frac{P}{A}, 20\%, 20\right)$$
$$+ 500000\left(\frac{P}{F}, 20\%, 20\right)$$
$$= ₹19,95,050$$

② Eg:

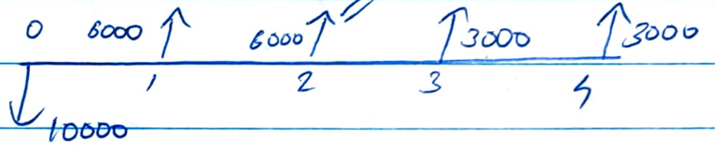


(A) $i = 18\%$

$$PW_A = -10000 + 3000\left(\frac{P}{F}, 18\%, 1\right) + 3000\left(\frac{P}{F}, 18\%, 2\right)$$

$$+ 7000\left(\frac{P}{F}, 18\%, 3\right) + 6000\left(\frac{P}{F}, 18\%, 4\right)$$

$$= ₹2052.1$$



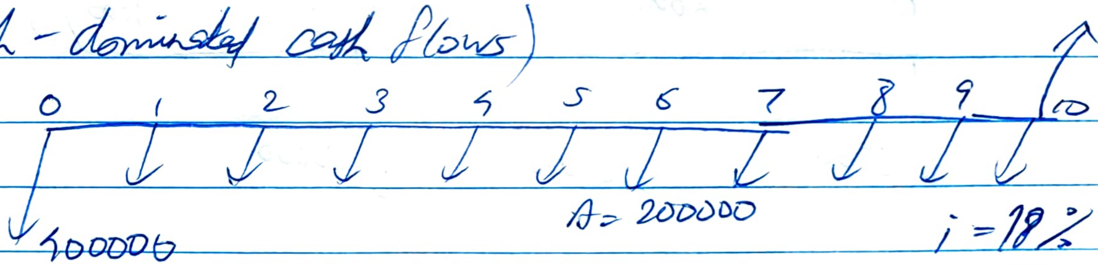
(B)

$$PW_B = -10000 + 6000\left(\frac{P}{F}, 18\%, 1\right) + 6000\left(\frac{P}{F}, 18\%, 2\right)$$

$$+ 3000\left(\frac{P}{F}, 18\%, 3\right) + 3000\left(\frac{P}{F}, 18\%, 4\right)$$

$$= ₹2767.4$$ (Worth more)

Ex:- ③ (Cost - dominated cash flows)



$i = 18\%$
$n = 10$

$$NPW_1 = -1600000$$

$$NPW_2 = -400000 - 200000\left(\frac{P}{A}, 18\%, 10\right)$$

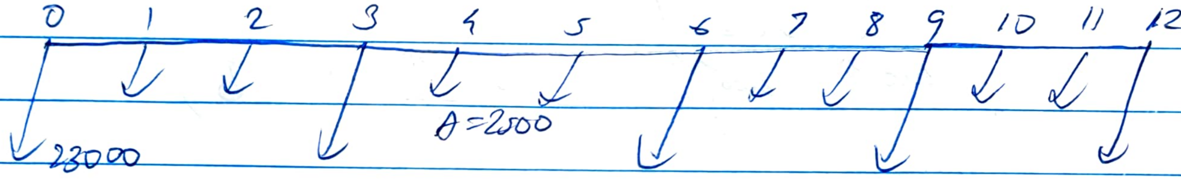
$$= -1298800$$ (Better)

— LCM method (for comparison of assets having unequal lives)

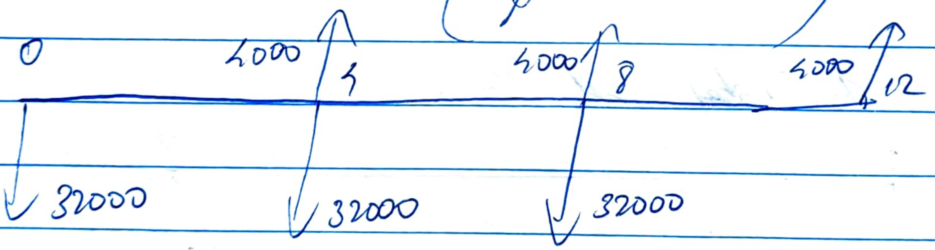
Sol:

A_1	A_2	
$P = 23000$	$P = 32000$	
$A = 2500$	$A = 0$	[LCM(3, 4)
$n = 3$	$n = 4$	= 12]
$F = 0$	$F = 4000$	

(A_1)



$$NPW_{A_1} = -23000 - 23000\left(\frac{P}{F}, 15\%, 3\right) - 23000\left(\frac{P}{F}, 15\%, 6\right) - 23000\left(\frac{P}{F}, 15\%, 9\right) - 2500\left(\frac{P}{A}, 15\%, 12\right) = -68156.8$$



$$NPW_{A_2} = -32000 - 28000\left(\frac{P}{F}, 15\%, 4\right) - 28000\left(\frac{P}{F}, 15\%, 8\right) + 4000\left(\frac{P}{F}, 15\%, 12\right)$$

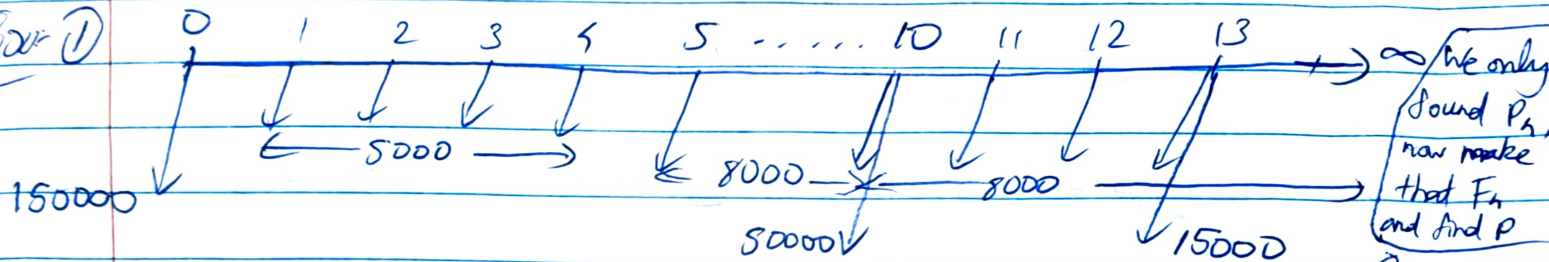
$$= -56416 \quad \text{(better)}$$

✯ Capitalized Cost

- Present Worth of a project that lasts forever ($n = \infty$)

$$\boxed{CC = \frac{A}{i}}$$

Ex-①



$$CC = -150000 - \frac{5000}{\cancel{0.05}}\left(\frac{P}{A}, 5\%, 4\right) - \frac{8000}{0.05}\left(\frac{P}{F}, 5\%, 4\right)$$

$$- 50000\left(\frac{P}{F}, 5\%, 10\right) - \frac{15000}{0.05}\left(\frac{A}{F}, 5\%, 13\right)$$

$= ₹ -347007$

~~annualized~~ here we convert this to F and find the equivalent A's as cost

Equivalent annual worth (AW) $= CC_{total} \times i$

$= ₹ -17350.35$

✯ Annual Worth

- All receipts and disbursements occurring over a period are converted to an equivalent uniform yearly amount.

- This way, it is possible to view a year's gains and losses as a milestone for progress.

- Capital recovery factor converts lumpsum into annuity.

- Consider $P \rightarrow$ first cost of asset.
  $F \rightarrow$ estimated salvage value at the end.
  $n \rightarrow$ estimated service life in years.
  $i \rightarrow$ interest rate or return.

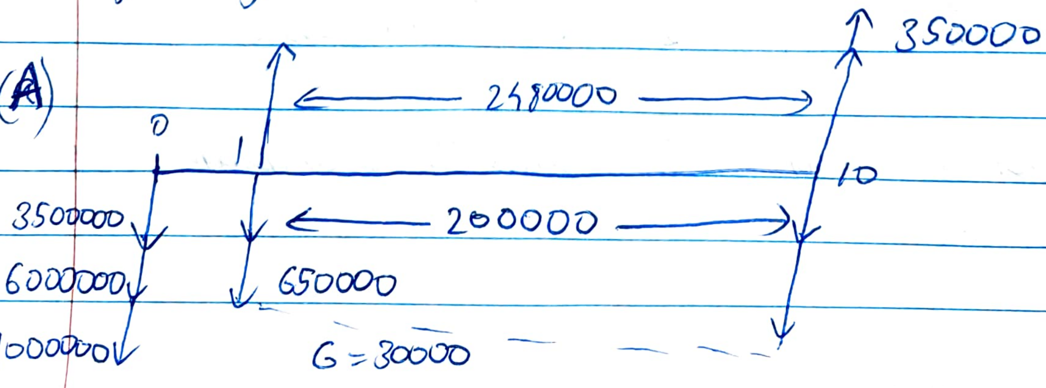
$$CR(i) = P(A/P, i, n) - F(A/F, i, n)$$
$$= (P-F)(A/P, i, n) + F.i$$

Ex: 1

Expense	Proposal	Proposal B	Proposal C
Building + Installation	6,000,000	7,000,000	4,000,000
Compressors	10,00,000	13,50,000	8,50,000
Expected energy cost / year	6,50,000	5,80,000	6,50,000
Cost increase	30000	20000	35000
Annual maintenance cost	200000	150000	500000
Salvage Value	350000	430000	180000

Plan A and B require to spend 35,00,000 on land while Plan C require 3500000, they all are expected to retain their value in 10 years. The estimated income increase due to facility available is annualized at ₹2480000/year.

(A)



(A) Income $= +2480000$

Initial Cost $= -35,00,000 - 6,000,000 - 1,000,000$

Annualized $= -1,05,00,000 \times (\frac{A}{P}, 10\%, 10)$

$= -17,08,875$

Salvage Value $= +3,50,000$

Annualized $= +350000 \times (\frac{A}{F}, 10\%, 10)$

$= +21,962.5$

Maintenance $= -2,00,000$

Energy cost $= -6,50,000 - 30000(\frac{A}{G}, 10\%, 10)$

Annualized $= -7,61,765$

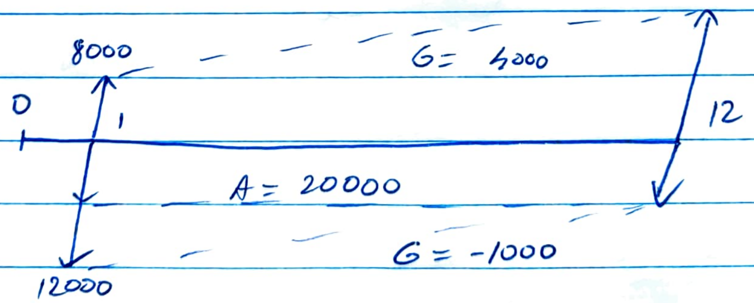
$AW_A = 2480,000 + 21962 - 1708875$
$- 200000 - 761765 = -1,68,677.5$

(B) (Do same)

$AW_B = -1,26,115$

(C) $AW_C = -3,10,810$

Ex-2



$i = 12\%$ compounded monthly

$= 1\%$ per month

Monthly Worth $= -20000 - 12000 + 1000(\frac{A}{G}, 1\%, 12)$

$+ 8000 + 4000(\frac{A}{G}, 1\%, 12)$

$= +2905$

Exr ③ (Case A)

Feature	A	B
Initial Cost	95000	251000
Operational Cost	19000	0
Life	4	8

$i = 8\%$

$AW_A = -95000\left(\frac{A}{P}, 8\%, 4\right) - 19000$

$= ₹ -47680$

$AW_B = -251000\left(\frac{A}{P}, 8\%, 8\right) - 0$

$= ₹ - 43680$ (Preffered)

(Case B)

$PW_A = -95000 - 19000\left(\frac{P}{A}, 8\%, 4\right)$

$-115000\left(\frac{P}{F}, 8\%, 4\right) - 4000\left(\frac{P}{A}, 8\%, 4\right)\left(\frac{P}{F}, 8\%, 4\right)$ (make P→F)

$= -2,52,192.47$

$AW = PW \times \left(\frac{A}{P}, 8\%, 8\right) = -43,884.01$

Internal Rate of Return (IRR)

- Discount rate that force Net Present Value to be zero.
- Percentage that indicates relative yield on different uses of capital.
- MARR (min acceptable rate of return) is the lowest level of return at which investment is acceptable ensuring best possible use of limited resources.
- To calculate, we must convert various consequences of the investment into cash flows, then solve for unknown IRR.
- PW of benefits = PW of costs

  $\frac{\text{PW of benefits}}{\text{PW of costs}} = 1$

  Net Present Worth = 0

  EUAB = EUAC (equivalent uniform annual worth)

Ex: $P = 8200$, $A = 2000$, $n = 5$

~~A = P(A/P)~~ $P = A\left(\frac{P}{A}, i\%, 5\right)$

(x_1) ~~10~~ 6 % $\Rightarrow P/A =$ ~~2.991~~ 4.212 (y_1)

x % $\Rightarrow P/A = 4.1$ (y)

(x_2) 8 % $\Rightarrow P/A =$ ~~1.863~~ 3.993 (y_2)

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\frac{x-6}{2} = \frac{-0.112}{-0.219}$$

$$x = 7.022 \simeq 7\%$$

Ex:-

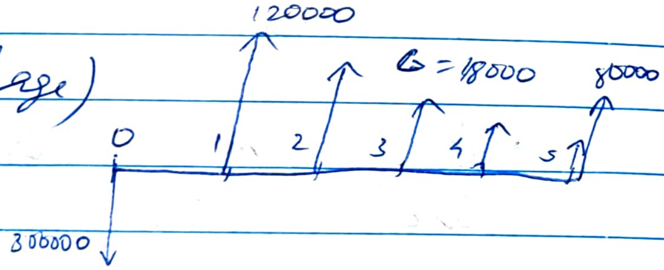
$G = -18000$ (15% of 120000)

$A1 = 120000$

$F = +80000$. (salvage)

$n = 5$

$P = 300000$



$$NPW = -300000 + 120000\left(\frac{P}{A}, i, 5\right) - 18000\left(\frac{P}{G}, i, 5\right) + 80000\left(\frac{P}{F}, i, 5\right)$$

	i	NPW	
(x_1)	25%	-26776	(y_1)
(x_2)	20%	2764	(y_2)
(x)	x%	0	(y)

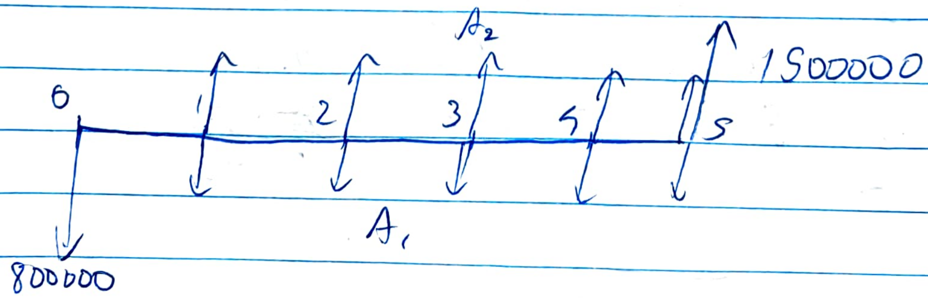
$$\frac{x-25}{-5} = \frac{26776}{29540}$$

$$x = 20.46\%$$

MARR = 20%

Since IRR > MARR, financially acceptable & profitable.

Ex:



$P = 800000$ $\quad A_2 = +15000$ $\quad n = 5$

$F = 1500000$ $\quad A_1 = -8500$

$$NPW = -800000 + 15000\left(\frac{P}{A}, i, 5\right) - 85000\left(\frac{P}{A}, i, 5\right) + 1500000\left(\frac{P}{F}, i, 5\right)$$

i	NPW
25%	-290975
12%	~~-177708.5~~ 745325
x%	0

$x = 14.65\%$

- **Incremental IRR**

• Examining differences between your alternatives to decide whether extra (differential) costs of more expensive option are actually justified by extra (differential) benefits.

Ex: $n = 20$, MARR $= 6\%$, No salvage value

	A	B	C
Initial Cost	2000	4000	5000
Uniform annual benefit	410	639	700

1. Compute IRR for each alternative individually
   A → 20%, B → 15%, C → 12.8%

② Since all ~~[illegible]~~ IRR > MARR, none of the alternatives are rejected.

③ Arrange alternatives in ascending order of initial cost

($A \longleftrightarrow B$) By convention, higher cost option (B), lower-cost option (A)

$$\Delta P \text{ (incremental cost)} = 4000 - 2000 = 2000$$
$$\Delta A \text{ (incremental benefit)} = 639 - 410 = 229$$

Now find IRR of diff ΔIRR_{B-A}

$$(P/A,\ \Delta IRR_{B-A},\ 20) = \frac{2000}{229} = 8.7336$$

$$\Delta IRR_{B-A} \approx 9.6\% \qquad (> MARR)$$

(Upgrade justified, B wins, A eliminated)

($B \longleftrightarrow C$)

$$\Delta P = 5000 - 4000 = 1000$$
$$\Delta A = 700 - 639 = 61$$

$$(P/A,\ \Delta IRR_{BC},\ 20) = \frac{1000}{61} = 16.393$$

$$\Delta IRR_{BC} \approx 2\% \qquad (< MARR)$$

(Upgrade not justified, C loses)

$\therefore$ Alternative B is the better/preferred option.

Replacement Analysis

- Defender: Current, old asset we are considering replacing
  Challenger: New asset proposed to take defender's place.

- Replacement of assets can lead to efficiency in operation as failure to upgrade assets can lead to serious loss in efficiency.

- Mass production is more economical for satisfaction of wants, only if machines used are financially viable.

- Additional expenses incurred for installation of a new machine before operation counts under initial cost.

— Sunk Cost

- Money already spent that cannot be recovered, does not affect future decisions involving replacement.

  Sunk Cost = Present Book Value − Present Market Value

(for unequal lives)

→ From the outsider's point of view, to compare the Equivalent Uniform Annual Cost (EUAC) between the Defender and Challenger, the one with lower EUAC wins

- For Defender, treat current market value as P (first cost)
  For Challenger, use purchase price as First Cost (P)

$$EUAC = P(A/P, i, n) + \text{Annual Operating Cost} - F(A/F, i, n)$$

where
P → first cost, F → salvage value.

- If removing an old machine costs money, that cost must be deducted from the recieved amount to get the Net Salvage Value.

— Cash Flow Approach (for assets with equal lives)

- When picking Challenger, the Defender's current market value is treated as a cash inflow (discount) on the Challenger's Price.

- Defender's first cost is taken as 0 and Defender market value is subtracted from Challengers' purchase price

- Capital Recovery Cost is the annual cost of owning the asset, accounting for loss in value & interest.

$$CR(i) = (P-F)(A/P, i, n) + F.i$$

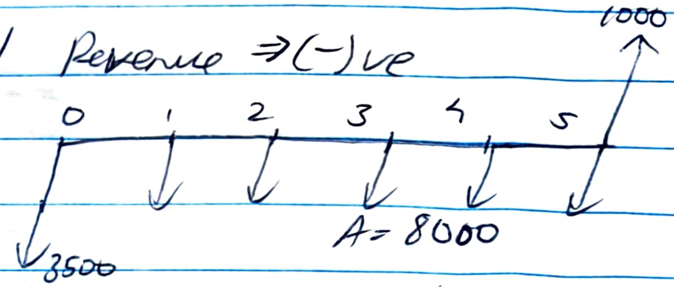
Ex:— Outsider Point of View

(a)

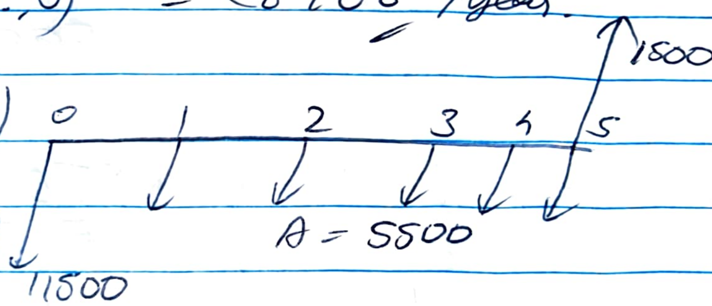
	Machine X (D)	Machine Y (C)
First Cost (P)	3500	11500
(AOC) Annual Operating Cost	8000	5500
Salvage Value (F)	1000	1500
Service life (n)	6-1 = 5 years	5 years

For machine X, we consider P as the current market value as per the salesman.

Assuming Cost ⇒ (+)ve and Revenue ⇒ (−)ve



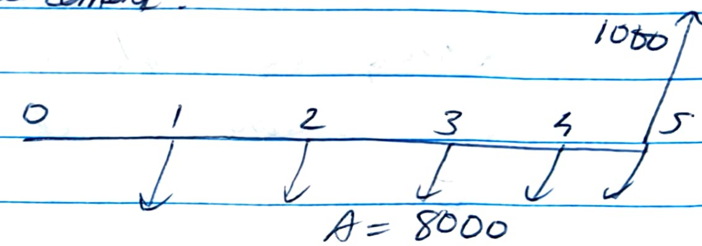
$$EUAC(X) = 3500\left(\frac{A}{P}, 8\%, 5\right) + 8000 - 1000\left(\frac{A}{F}, 8\%, 5\right) = ₹8706\ /year$$



$$EUAC(Y) = 11500\left(\frac{A}{P}, 8\%, 5\right) + 5500 - 1500\left(\frac{A}{F}, 8\%, 5\right)$$
$$= ₹8124.60\ /year$$

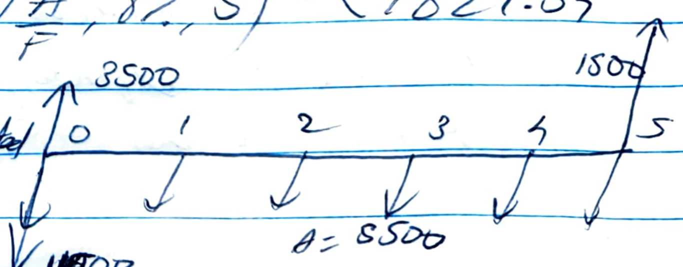
∴ It is more economical to replace machine X with Y thus saving ₹581 /year by replacement.

(b) <u>Cash Flow Approach</u>



Here Machine X will have an initial cost (P) of 0.

$$EUAC(X) = 8000 - 1000\left(\frac{A}{F}, 8\%, 5\right) = ₹7829.54$$



Here Machine Y has an discounted initial cost from Machine X.

$$EUAC(Y) = (11500 - 3500)\left(\frac{A}{P}, 8\%, 5\right) + 5500 - 1500\left(\frac{A}{F}, 8\%, 5\right)$$
$$= ₹7248\ /year.$$

∴ More economical to replace X with Y, saving ₹581.55/year

(c) If Challenger's (Machine Y) life is 10 years, determine best course of action.

$EUAC(X) = ₹8706.15/year \quad (n=5)$ (same)

$EUAC(Y) = 11500(A/P, 8\%, 10) + 5500 - 1500(A/F, 8\%, 10) = ₹7110.3/year$

$\therefore$ Savings $= ₹1595.85/year$.

(d) $EUAC(X) = ₹8706.15/year$

$EUAC(Y) = ₹8124.60/year$

Let present book value of Machine X = ₹7250

Sunk Cost of Machine X = Present Book Value − Present Market Value

$= 7250 - 3500 = ₹3750$

Adding the sunk cost of Machine X to first cost of Machine Y (challenger)

$EUAC(Y) = (11500 + 3750)(A/P, 8\%, 5) + 5500 - 1500(A/F, 8\%, 5) = ₹9063.83$

$\therefore$ It is economical to continue with the service of machine X (Defender) rather than replacing with Y,

Annual savings $= ₹357.68/year$.

Ex

	Old Truck (D)	New truck (C)
P	7500	11000
SV	2000	2000
ADC	3000	1800
n	3	10

$EUAC(Old) = 7500(\frac{A}{P}, 10\%, 3) + 3000 - 2000(\frac{A}{F}, 10\%, 3)$
$= ₹5411.6$ /year

$EUAC(New) = 11000(\frac{A}{P}, 10\%, 10) + 1800 - 2000(\frac{A}{F}, 10\%, 10)$
$= ₹3464.75$ /year

∴ It is economical to replace the old truck,
Annual savings = ₹1946.85 /year.

Present book value of old truck = ₹8100
Sunk cost = 8100 − 7500 = ₹600
∴ P for new truck = 11000 + 600 = ₹11600

$EUAC(\text{new truck}) = 11600(\frac{A}{P}, 10\%, 10) + 1800 - 2000(\frac{A}{F}, 10\%, 10)$
= ~~97~~ ₹3562.40 /year.

∴ Same decision,
Annual savings = ₹1849.2 / year.

Ex:

	Current Asset (D)	Challenger 1	Challenger 2
First Cost (P)	0	30000	54000
Defender trade-in	–	10500	7500
Annual Cost (AOC)	9000	4500	3600
Salvage Value	1500	3000	1500
Life (n)	5	5	5

Since equal life, we will use Cash Flow Approach.

$EUAC(D) = 9000 - 1500(\frac{A}{F}, 18\%, 5)$
$= ₹8790.33$ /year

$$EUAC(C1) = (30000 - 10500)\left(\frac{A}{P}, 18\%, 5\right) + 4500 - 3000\left(\frac{A}{F}, 18\%, 5\right)$$
$$= ₹10316.7 \text{ /year}$$

$$EUAC(C2) = (54000 - 7500)\left(\frac{A}{P}, 18\%, 5\right) + 3600 - 1800\left(\frac{A}{F}, 18\%, 5\right)$$
$$= ₹18261 \text{ /year}$$

$\therefore$ Since Defender has lowest EUAC, it is the best course of action to continue using it.

- Economic Life

- Specific time period that minimizes the asset's EUAC (optimum time for replacement)

- Balance between Capital Recovery Cost (Ownership) which goes down the longer you keep the machine as initial cost is spread over the years. and Annual Operating Cost (AOC) which goes up every year as machine gets older, breaks down and becomes less efficient

- Economic Life = Sum (Capital Recovery Cost, AOC) at lowest.

Eg: $i = 10\%$, Present Market Value (D) = ₹130000
$n = 5$

$$CR(i) = (P - F)\left(\frac{A}{P}, i, n\right) + F \cdot i$$

Life (n)	Salvage Value	AOC
1	90000	25000
2	80000	27000
3	60000	30000
4	20000	35000
5	0	45000

$CR(1) = (130000 - 90000)(A/P, 10\%, 1) + (90000 \times 0.1)$
$= ₹58000$

$EAOC(1) = ₹25000$

$EAOC(2) = [25000(P/F, 10\%, 1) + 27000(P/F, 10\%, 2)](A/P, 10\%, 2)$
$= ₹25952$

$EAOC(3) = [25000(P/F, 10\%, 1) + 27000(P/F, 10\%, 2) + 30000(P/F, 10\%, 3)](A/P, 10\%, 3)$
$= ₹27174$

$\therefore$

Year	CR(i)	+ AOC	= EUAC
1	58000	25000	78000
2	36810	25952	62762
3	34148	27174	61322
4	36702	28861	65563
5	34294	31504	65798

$\therefore$ Minimum total EUAC occurs at year 3

$\therefore$ Economic life of asset = 3 years.

Eco	Years	Operating Cost (+50000)	Maintenance Cost (+5000)	Salvage Value (-50000)
	1	100000	10000	500000
	2	150000	15000	450000
	3	200000	20000	400000
	4	250000	25000	350000
	5	300000	30000	300000
	6	350000	35000	2500000

~~AE(1) = P~~

P = ₹1,000,000/-, F = Salvage Value, i = 15%

$AE(1) = (P-F)(\frac{A}{P}, 15, 1) + F.i + OC + MC$

$= (1000000 - 500000)(\frac{A}{P}, 15\%, 1) + (500000 \times 0.15)$
$+ 100000 + 50000(\frac{A}{G}, 15\%, 1)$
$+ 10000 + 5000(\frac{A}{G}, 15\%, 1)$
$= ₹760000$

AE(2) = ₹ 541325

AE(3) = ₹ 482685

AE(4) = ~~₹ 458310~~ ₹ 463125

AE(5) = ₹ 458310

↑↑↑ AE(6) = ₹ 460835

∴ Economic life = 5 years

Depreciation

- Loss of value for a fixed asset over time. ~~Business~~ Business Expense that reduces taxable income

- Taxable Income = Gross Income − Expenses (including depreciation & cost of goods sold)

Taxable Income − Income Tax = Net income (profit)

- Causes: Wear & Tear during operation, inability to meet demand, assets getting outdated, accidents, misuse, inherent technical failure, depletion of assets.

- Factors: Depreciable life (n) (how long asset will be useful)
  Salvage Value (F) (estimated scrap value at EOL)
  Cost Basis (P) (initial cost of asset)
  Method of Depreciation (approach used to spread cost over time.)

— Straight-Line Method

- Assuming asset loses an equal amount of value every year.

Annual Depreciation, $D_t = \dfrac{P-F}{n}$

Book Value at time t, $B_t = P - t\left(\dfrac{P-F}{n}\right)$

Depreciation Rate, $= \dfrac{1}{n}$

Eg:

t	D_t	B_t
0	—	$P = 5000$
1	$\frac{P-F}{n} = 800$	$P - \left(\frac{P-F}{n}\right) = 4200$
2	800	$P - 2\left(\frac{P-F}{n}\right) = 3400$
3	800	2600
4	800	1800
5	800	$1000 = F$

Eg: $P = 10000$, $N = 5$, $F = 2000$, $D = \frac{P-F}{N} = 1600$

t	D_t	B_t
0		10000
1	1600	8400
2	1600	6800
3	1600	5200
4	1600	3600
5	1600	$2000 = F$

Eg: $t = 8$, $P = 500000$, $F = 50000$

Book Value, $B_8 = P - 8\left(\frac{P-F}{n}\right)$

$$B_8 = 500000 - 8\left(\frac{500000 - 50000}{10}\right) = ₹140000$$

Book Value > Market Price

Sunk Loss $= 140000 - 90000 = ₹50000$

—x—

- **Declining Balance Method**

- Assumes asset value decreases faster in early stages of life than in later stages. (R = Depreciation Rate)

$$\boxed{D_t = R(1-R)^{t-1}P} \qquad \boxed{B_t = (1-R)^t P}$$

$$\boxed{1-R = \left(\frac{B_t}{P}\right)^{\frac{1}{t}}} \qquad \boxed{D_t = R \times B_{t-1}}$$

Year	D_t	B_t
0	–	$P = B_0$
1	$R\times B_0 = R\times P$	$B_0 - R\times B_0 = (1-R)B_0 = B_1$
2	$R\times B_1 = R(1-R)P$	$(1-R)B_1 = (1-R)^2 B_0 = B_2$
3	$R\times B_2 = R(1-R)^2 P$	$(1-R)B_2 = (1-R)^3 B_0 = B_3$
4	$R(1-R)^3 P$	$(1-R)^4 B_0 = B_4$
$\vdots$	$\vdots$	$\vdots$
t	$R(1-R)^{t-1}P$	$(1-R)^t B_0 = B_t$
$\vdots$	$\vdots$	$\vdots$
n	$R(1-R)^{n-1}P$	$(1-R)^n B_0 = B_n$

Ex: $P = 5000$, $F = 1000$, $n = 5$, $R = 0.3$ (30%)

Year	D_t	B_t
0	–	5000
1	$5000\times 0.3 = 1500$	$5000 - 1500 = 3500$
2	$3500\times 0.3 = 1050$	$3500 - 1050 = 2450$
3	$2450\times 0.3 = 735$	$2450 - 735 = 1715$
4	$1715\times 0.3 = 514.5$	$1715 - 514.5 = 1200.5$
5	$1200.5\times 0.3 = 360.15$	$1200.5 - 360.15 = 840.35$

∴ At the end of 5 years service life, total depreciation charges are as per table above.
Final book value $= ₹840.35$

Eg: $P = 250000,\ n = 10,\ F = 50000$

$$R = 1 - \left(\frac{F}{P}\right)^{\frac{1}{n}} = 1 - \left(\frac{50000}{250000}\right)^{\frac{1}{10}} = 0.1487$$

(14.87%)

$$B_6 = (1-R)^6 \times P$$
$$= ₹96180$$

$$D_8 = R(1-R)^7 P = ₹12163$$

— Double-Declining Balance Method

- Here depreciation rate is twice the straight-line rate, $R = 2\left(\frac{1}{n}\right) = \frac{2}{n}$

Eg: $P = 500000$, $F = 50000$
$n = 20$, $t = 10$, $R = \frac{2}{n} = 0.1$

$$B_{10} = (1-R)^{10} P = ₹174339 > F \quad \text{(Valid)}$$

Eg: $P = 48000$, $n = 20$, $t = 5$, $R = \frac{2}{n} = 0.1$

$$B_5 = (1-R)^5 P = ₹28343.52$$

Total Accumulated Depreciation $= P - B_5$
$= 48000 - 28343.52$
$= ₹19656.48$

Break-Even Analysis

- For a business to survive, it must make a profit

Profit: Occurs when Total Income (Revenue) > Total Expenditure (Costs)

Loss: Occurs when Total Cost > Total Income.

- Fixed Cost (FC) must be paid even if no products are sold (rent, insurance, interest on loans)

- Variable Cost (VC) increase with every extra product sold (raw materials, labor, energy)

Total Cost = Fixed Cost (FC) + Variable Cost (VC)

- Break-even Point is a specific level of sales when total costs = total sales, Profit = 0, Loss = 0.

- Any sales above BEP → Profit

Any sales below BEP → Loss

- $$BEP = \frac{\text{Fixed Cost}}{\text{Selling Price per unit} - \text{Variable Cost per unit}}$$

- Contribution Margin (TCM): Marginal profit per unit sale.

(TCM = TR − TVC)

- Average Contribution Margin (ACM): Unit price minus Average Variable Cost

(Break-even occurs when ACM = AFC)

- Unit CM is the amount each unit sale adds it profit

- Assumptions: (i) Costs are either fixed / variable
  (ii) SP of products remains constant regardless of volume
  (iii) VC increases at a constant rate
  (iv) No change in technology or labor efficiency
  (v) Production - Sales sync (no inventory buildup)

- Useful for determining min. output level required to avoid a loss, deciding whether to make parts in-house or buy from suppliers, expand / contract a plant, etc.

Ex: $SP = ₹5/-$ $FC = ₹60000$
$VC = ₹2/-$

(a) $$BEP = \frac{FC}{SP - VC} = 20000 \text{ units}$$

(b) To make profit, Income > Expenditure.
$TR = 30000 \times 5 = ₹150000$
$VC = 30000 \times 2 = ₹60000$
$\text{Total Cost} = FC + VC = 60000 + 60000 = ₹120000$

$\text{Profit} = TR - TC = 150000 - 120000 = ₹30000$

(c) If firm spends ₹3000 on ads,
$\text{New } FC = 60000 + 3000 = ₹63000$
$$\text{New } BEP = \frac{63000}{5-2} = 21000 \text{ units}$$

(d) To make profit of ₹30000
$$\text{Required Units} = \frac{FC + \text{Target Profit}}{SP - VC} = \frac{60000 + 30000}{5-2} = 30000 \text{ units}$$

Ex: $i = 10\%, \ n = 4$

	(Electric) Alternative A	(Diesel) Alternative B
First Cost (P)	4900	1925
Salvage Value (F)	700	0
Maintenance	420/year	Included
Operating Cost	2.94/hour	1.47 + 0.53 + 2.8 = 4.80/hour

~~EUAC = P(A/P, i, n) − F~~

$$EUAC(A) = \left[P\left(\frac{A}{P}, i, n\right) - F\left(\frac{A}{F}, i, n\right)\right] + 420 + 2.94H$$

$$= \left[4900\left(\frac{A}{P}, 10\%, 4\right) - 700\left(\frac{A}{F}, 10\%, 4\right)\right] + 420 + 2.94H$$

$$EUAC(B) = 1925\left(\frac{A}{P}, 10\%, 4\right) + 4.8H$$

At break-even point, $EUAC(A) = EUAC(B)$

$H = 649.3$ hours/year

Now let $H = 100$ hours/year.

$EUAC(A) = ₹2108.97$/year

$EUAC(B) = ₹1087.28$/year (more economical)

Ex:

	Gasoline	Diesel	Butane
First Cost (P)	2000	2800	3300
Maintenance Cost	200/year	240/year	375/year
Fuel Cost	3.6/hour	3.3/hour	2.9/hour

$F = 0$ (all), $i = 15\%, \ n = 5$

Annual Cost of Gasoline

$$AC_G = 2000\left(\frac{A}{P}, 15\%, 5\right) + 200 + 3.6H$$

$$AC_D = 2800\left(\frac{A}{P}, 15\%, 5\right) + 240 + 3.8H$$

$$AC_B = 3300\left(\frac{A}{P}, 15\%, 5\right) + 315 + 2.9H$$

If $AC_G = AC_B \Rightarrow H = 718.27$ hours/year
If $AC_D = AC_B \Rightarrow H = 560$ hours/year
If $AC_G = AC_D \Rightarrow H = 928$ hours/year.

$\therefore$ 0 – 718 hours/year $\Rightarrow$ Gasoline Engine
(lowest starting cost, economical for low usage)
718 hours/years ++ $\Rightarrow$ Butane Engine
(Cheapest per hour of usage which will outweigh it's high upfront cost) (Diesel does not cross Gasoline until 929 hours, making it the worst choice)

~~A~~ Financial Statements & Analysis

- Our goal is to understand financial reports, presented as Annual Reports, which consist of a Verbal section and Final Accounts section to monitor business progress, profit and loss and assess financial soundness at the right time.

– Trading Account

- Debit Side: Records all Direct Expenses (Opening Stock, Purchases, Wages, Freight, Power)
- Credit Side: Records all Direct Incomes (Sales, Closing Stock)

- ~~All~~ Balance on Debit side indicate Gross Profit
  Balance on Credit side indicate Gross Loss

– Profit-and-Loss Statement

- Debit Side: Records all Indirect Expenses (office admin, selling distribution, financing, maintenance expenses)
- Credit Side: Records all Indirect Incomes (discounts, commission, interest recieved)
- Balance on Debit Side indicate Net Profit
  Balance on Credit Side indicate Net Loss.

– Balance Sheet

~~It is~~ Statement that sets out Assets and Liabilities of a firm to ascertain it's financial position on a specific date

Liabilities

- Obligations the business owes to outside parties
- Current / Short-Term (loans, salaries, taxes payable)
- Long-term (debts, accrued liabilities)
- Owner's Equity (Share Capital) (Reserves & Surplus)

Assets

- Resources owned by the business, categorized by liquidity & physical form
- Current / Short-term (Cash, Demand Deposits)
- Fixed / Long-term Assets (land, plant, equipment)
- Tangible; physical assets that undergo depreciation
- Intangible; non-physical assets like brand-name, patents, copyrights.

Financial Ratio Analysis

- Index that relates two accounting numbers to evaluate a firm's health and performance.

① Liquidity Ratio (Measures ability to meet short-term obligations)

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

$$\text{Acid Test / Quick Ratio} = \frac{\text{Current Assets} - \text{Inventories}}{\text{Current Liabilities}}$$

② Financial Leverage Ratio (extent to which firm uses borrowed money)

$$\text{Debt-To-Equity} = \frac{\text{Total Debt}}{\text{Shareholder's Equity}}$$

Debt-to-Total Assets measures financial risk relative to total assets.

③ Turnover Ratios (measure efficiency in asset management and how quickly assets convert into sales)

$$\text{Inventory Turnover} = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory}}$$

$$\text{Debtors Turnover} = \frac{\text{Net Credit Sales}}{\text{Average Debtors (Accounts Recievable)}}$$

$$\text{Average Collection Period} = \frac{\text{Average Debtors}}{\text{Credit Sales}} \times 360 \text{ days}$$

(how slow collection is)

④ Profitability Ratios (measure overall efficiency and public acceptance of products)

$$\text{Gross Profit Margin} = \left(\frac{\text{Gross Profit}}{\text{Sales}}\right) \times 100$$

$$\text{Return on Investment (ROI)} = \frac{\text{Net Profit After Tax}}{\text{Average Total Assets}}$$

Return on Equity (ROE) (measures utilization of owner's funds)

$$= \frac{\text{Net Profit After Tax}}{\text{Average Shareholders Equity}}$$

$$\text{Net Profit Margin} = \frac{\text{Profit after Tax (EAT)}}{\text{Sales}}$$

Note: Cost of Goods Sold = Opening Stock (inventory sitting at start)
+ Manufacturing Cost (direct expenses)
− Closing Stock (inventory remaining)

Note: $$\text{Interest Coverage Ratio} = \frac{\text{Earning before Interest \& Taxes}}{\text{Interest Expense}}$$

(Larger the ratio, greater the ability of the firm to handle fixed charge liabilities.)

Note: Average Inventory = Avg of monthly inventory for calendar year

$$= \frac{\text{Opening Stock} + \text{Closing Stock}}{2}$$

Ex:- Credit Sales = ₹24L

Opening ~~Avg~~ debtors = ₹2.8L

Closing ~~Avg~~ bills recievable = ₹3.2L

Average Debtors = $\frac{2.8+3.2}{2}$ = ₹3L

Debtor Turnover ratio = $\frac{24}{(2.8+3.2)/2} = 8$ times

~~Avg collection period = $\frac{2.8+3.2}{24} \times 360$~~

Average collection period = $\frac{\text{Average Debtors}}{\text{Credit Sales}} \times 360$

$$= \frac{3}{24} \times 360 = 45 \text{ days}$$

Eg: Total Sales = ₹3L, Gross Profit Margin = 25%

Current Liability = ₹65000

Inventory = ₹34000, Cash = ₹13000

Cost of Goods Sold = (1 − GPM) × Total Sales

(Cost of making goods is 75% of sales)

= 0.75 × 300000 = ₹225000

Inventory Turnover = $\frac{\text{Cost of Goods Sold}}{\text{Avg Inventory}} = 5$

~~Average~~ Avg Inventory = $\frac{225000}{5}$ = ₹45000

Current Assets = 2.5 × Current Liabilities

= ₹162500

Closing Debtors = ₹162500 − 34000 − 13000

= ₹115500

$$\text{Avg Debtors} = \frac{\text{Opening} + \text{Closing}}{2} = \frac{50000 + 115500}{2} = ₹82750$$

$$\text{Avg Collection Period} = \frac{\text{Avg Debtors}}{\text{Total Credit Sales}} \times 365$$

$$= \frac{82750}{300000} \times 365 = 101 \text{ days}$$

—x—